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A theoretical investigation on the spectroscopy and structure of the exotic tetraquark states

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Abstract

Treating the tetraquark as a composite system of diquark-antidiquark, the effective masses of diquarks have been estimated in the Composite Fermion (CF) quasiparticle model. Hence, the masses of the exotic tetraquark states such as T_{cc}, Z_c, X, Y, f_0 etc. have been explored in the framework of the Composite Fermion (CF) approach and the computed results compare favourably with the experimental data. Moreover, the form factors of these exotic tetraquarks have been investigated for the momentum transfers, Q^2 in the range of $0.01 < Q^2 < 1.0$ GeV^2. The form factors are found to decrease sharply with the increase of Q^2 in the range of $0.01 < Q^2 < 0.1$ GeV^2 which indicates the scaling violation with the gluon contribution to the form factor and the structure function. However, the form factors show gradual decrease with a small scaling violation in the range of $0.1 < Q^2 < 1.0$ GeV^2.

Keywords: Tetraquarks; Multihadron states; Diquarks; Form factor

1. Introduction

One of the most challenging area is the understanding of the exotic hadron spectroscopy, structure and dynamics in the frame work of QCD and beyond. In recent years, a number of mul-

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tiquark states have been discovered in high energy experiments which cannot be described by usual baryon (qqq) or meson ($q\bar{q}$) configurations. Since the discovery of $X(3872)$ by Belle group [1], a number of tetraquark states have been reported like $Y(3940)$ [2], $Y(4140)$ [3], $Z^{+}(3940)$ [4], $Z_c^{+}(4200)$ [5] which are charmonium like states and $Z_b(10610)$, $Z_b(10650)$ [6] which are bottomonium like states. Recently, LHCb [7] has reported an observation of tetraquark structure $X(6900)$, while studying the mass spectrum of J/Ψ pairs using the proton-proton collision data at the centre of mass energies of $\sqrt{S} = 7.8$ and 13 TeV. LHCb [8] has announced the discovery of three additional tetraquark candidates X(4274), X(4500) and X(4700). A number of models and approaches have been proposed by different authors to study the tetraqurk states. Jaffe [9] has studied the spectra and dominant decay couplings of multiquark hadrons $Q^2\bar{Q}^2$ in the framework of quark bag model. The tetraquark state T_{4c} was first proposed by Iwasaki [10] after the discovery of J/Ψ meson. Debastiani et al. [11] have studied the tetraquark masses both in diquark-antidiquark approach and meson molecule. Chen et al. [12] have investigated doubly hidden charm and bottom masses in diquark-antidiquark configuration for different J^{PC} states and have observed that the masses are higher than the observed spontaneous dissociation threshold of two charmonium mesons while performing the QCD sum rules. Wang et al. [13] have studied the mass spectra of s-wave fully heavy tetraquark states in diquark-antidiquark picture in non-relativistic quark model with one gluon exchange Coulomb linear confinement type potential and hyperfine interaction between the diquark and antidiquark. Doubly heavy tetraquark masses have been investigated by a number of authors [14–18] in the context of the constituent quark models and QCD sum rules. Chakrabarti et al. [19] studied the multiquark states with di-hadronic states which also reproduced the masses within the experimental predictions.

The idea of the electromagnetic form factor comes from the extended structure of hadrons and deviation from the point particle behaviour which also gives the idea of charge distribution of the system. Regarding tetraquarks, the extraordinary properties of the first discovered exotic state $X(3872)$ [1], such as isospin violation [20] and radiative decays [21,22] still make its structure unsolved. The structures of these multiquark states need a detailed study. Information about the structure of hadrons is obtained from the phenomenological experiments on electron hadron elastic scattering and electron-positron annihilation to hadron-antihadron pairs. The electromagnetic form factor as a function of momentum transfer gives us a complete description of the structure. The study of the properties like moments, hadronic amplitude, form factors are crucial for understanding the hadron properties and it's resonances. The large contribution of magnetic moment of proton, anomalous magnetic moment of proton and anomalous magnetic moment of the neutral particle neutron demand the extended structure. It is noteworthy to mention a few works of the authors [23–25] on the electromagnetic form factors of hadrons. The study of the form factors with the momentum transfer gives us the idea of gluon contribution to the system and scaling violation. Discovery of approximate scaling in deep inelastic scattering indicates the parton distribution and corresponding model of hadrons. Faustov et al. [26] have made a review on the heavy tetraquark states in relativistic quark model. They have studied the interanl structure of the tetraquarks by calculating the form factor of diquark-gluon interaction. Agaev et al. [27] have studied doubly heavy axial vector tetraquarks decay using QCD sum rules. They have extracted the weak form factors of these exotics from semileptonic and partial decay width rate from three point QCD sumrule. Gerasyuta et al. [28] have calculated the electromagnetic form factor of multiquark states with N quarks based from transition from Feynman amplitude to the dispersion integration over the masses of composite particles. Doubly heavy tetraquarks have been thoroughly investigated by a number of workers such as Lu et al. [29], Mutuk [30], Qin et al. [31], Francis et al. [32]. Excellent reviews on tetraquark states have been done by Lucha et al. [33], Olsen et al. [34], Guo et al. [35], Chen et al. [36]. The dynamics of fully or partly heavy tetraquarks are under extensive study. More experimental observations are needed to understand the structure and the spectra of these tetraquarks.

In the present work we have estimated the masses of the tetraquark states in the Composite Fermion approach. Using the diquark masses, the masses of the exotic tetraquark states such as T_{cc}, Z_c, X, Y, f_0 etc. have been calculated. We have also investigated in detail the form factors of these tetraquark states for momentum transfers Q^2 in the range of $0 < Q^2 < 0.1$ GeV^2 and $0.1 < Q^2 < 1.0$ GeV^2 separately. Variations have been shown in the graphs and studied in detail. Results have been compared with the other theoretical and experimental values.

2. Formulation

2.1. Estimation of the tetraquark mass in the diquark-antidiquark configuration

We have suggested a Composite Fermion model for the diquark in an analogy with the Composite Fermion (CF) picture in the usual Quantum Hall effect. Diquark behaves like a Composite Fermion in the presence of the chromo-magnetic behaviour of QCD vacuum. The effective mass of the Composite Fermion can be determined in a gauge invariant way and the leading contribution comes in the limit $\frac{\Lambda}{p_F}$, where Λ is cut off parameter and p_F is the Fermi momentum of the Composite Fermion. The effective mass of the diquark as CF can be computed in a gauge invariant way. It is to be mentioned that Chari et al. [37] proposed an effective mass of quasiparticle in a gauge invariant way and expressed it in terms of response function of the system in analogy with Landau's picture of quasiparticle of Fermi liquid. Starting from the Hamiltonian of a Composite Fermion with a momentum cut-off Λ, the expression for the quasi particle mass in gauge invariant system can be expressed as (with potential V=0):

$$\frac{1}{m^*} = \frac{1}{m}(1 + \frac{\Lambda^4}{2p_F^4}) \tag{1}$$

where m^* is the effective mass of the CF, m is the constituent mass, p_F is the Fermi momentum of the CF and Λ is cut off parameter. The CF picture has been applied in computing the diquark mass. The effective mass of the diquark now can be expressed as following (1),

$$\frac{1}{m_D^*} = \frac{1}{(m_q + m_{q'})}(1 + \frac{\Lambda^4}{2p_F^4}) \tag{2}$$

The Fermi momentum of mesons of different flavours of diquark have been computed using the work of Bhattacharya et al. [38,39], where a relation between the Fermi momentum and the radius of the corresponding meson has been derived. We have estimated the Fermi momentum of diquarks of different flavours with the input of diquark radius from the existing literature [40–44] and computed the masses of diquarks using relation (2). With $m_u = m_d = 0.360$ GeV, $m_s = 0.540$ GeV, $m_c = 1.71$ GeV, $m_b = 5.05$ GeV [45], $\Lambda = 0.573$ GeV for light sector [46] and 0.6533 GeV for heavy sectors [47], we have estimated the masses of scalar and vector diquarks of different flavours which have been displayed in Table 1. It is to be noted that the effective mass of the diquark may be less than the combined mass of the constituent quarks, since it has been treated as CF in the presence of chromo-magnetic behaviour of QCD vacuum. The surrounding field is responsible for the mass coagulation.

We have assumed that the mass of the antidiquark can be estimated using the same formalism as the diquark. It may be mentioned that when a collective excitation occurs in a complicated microscopic system, the formulation is equally applicable for the antidiquarks also. The diquark and the antidiquark masses of same flavour possess the same effective mass in the current approach.

The mass formula for the tetraquark state in the relevant diquark-antidiquark configuration is simply additive with binding energy and spin interaction between them which runs as,

$$M = m_D + m_{D'} + E_{BE} + E_S \tag{3}$$

where m_D and $m_{D'}$ are the effective masses of diquark and antidiquark respectively, E_{BE} is the binding energy of the diquarks and E_S is the spin term. The binding energy of the diquarks can be expressed in the form of potential as,

$$E_{BE} = < \Psi(r_{12})|V(r)|\Psi(r_{12}) > \tag{4}$$

where $\Psi(r_{12})$ is the wavefunction of the tetraquark state and r_{12} is the radius parameter of the tetraquark. $V(r)$ is assumed to be of linear type of potential between diquark and antidiquark which is stated as,

$$V(r) = ar \tag{5}$$

To estimate E_{BE}, we have used the wave function for the ground state of the tetraquark from the Statistical Model [38],

$$|\Psi(r_{12})|^2 = \frac{315}{64\pi}(r_{12})^{-9/2}(r_{12}-r)^{3/2}\Theta(r_{12}-r) \tag{6}$$

where $\Theta(r_{12}-r)$ is the usual step function and $r_{12} = r_1 + r_2$, r_1 and r_2 representing the individual radius of diquark and antidiquark respectively. The spin term is expressed as,

$$E_S = \frac{8}{9}\frac{\alpha_S}{m_D m_{D'}} S_1.S_2|\Psi(0)|^2 \tag{7}$$

where $S_1.S_2$ is the spin interaction of the corresponding states. The spin components have been taken into account through the combinations of scalar and vector diquark-antidiquark. With the values of $a = 0.07$ GeV^2, $\alpha_s = 0.2$ [48], the ground state masses of the exotic tetraquark states for different combinations of scalar and vector diquark-antidiquark have been estimated using the relation (3) and are reported in the Table 2. The computed masses have also been compared with the corresponding experimental values [3,7,49–53].

2.2. Estimation of the form factors of the tetraquarks

To study the structure of the tetraquarks, we would like to investigate their electromagnetic form factors in detail which describes the spatial distribution of charges within the hadron. The expression of the form factor can be written as:

$$F(Q^2) = \int |\Psi(r)|^2 e^{i.Q.r}.dr \tag{8}$$

Q being the momentum transfer and r the radius vector with the origin at the center of the scattering system. In estimating the form factor of the tetraquark state we have used the wave function for hadrons derived in the context of Statistical Model [38,39]. The wavefunction is described as:

Table 1
Diquark masses computed in CF model.

Diquark Content $[qq]$	Scalar Diquak Mass in GeV	Vector Diquark Mass in GeV
$[ud]$	0.4848	0.5957
$[us]$	0.4287	0.5913
$[uc]$	0.9158	1.4052
$[sc]$	1.0363	1.6958
$[ub]$	3.568	4.0775
$[sb]$	4.0244	4.5258
$[cc]$	3.124	3.1531
$[bc]$	6.4575	6.4599
$[bb]$	9.9712	9.9811

Table 2
Ground state Tetraquark masses in GeV computed for different combinations of scalar and vector diquark-antidiquark.

Tetraquark	Computed Mass in GeV (Quark Content)				Expt. Mass in GeV
	$[qq]_0[\bar{q}\bar{q}]_0$	$[qq]_0[\bar{q}\bar{q}]_1$	$[qq]_1[\bar{q}\bar{q}]_0$	$[qq]_1[\bar{q}\bar{q}]_1$	
$f_0[ud][\bar{u}\bar{d}]$	1.3439	1.4205	1.4205	1.4985	1.370 [53]
K$[ud][\bar{u}\bar{s}]$	1.3322	1.4554	1.4087	1.5329	1.460 [53]
D$[cd][\bar{d}\bar{s}]$	1.7861	1.9093	2.2298	2.3535	2.633 [51]
$T_{cc}[ud][\bar{c}\bar{c}]$	3.9059	3.9320	3.9825	4.0090	3.875 [52]
$Z_s[uc][\bar{s}\bar{c}]$	2.3685	2.9710	2.8122	3.4156	4.003 [50]
$Z_c[cd][\bar{c}\bar{u}]$	2.2518	2.6956	2.6956	3.1395	3.900 [49]
Y$[cs][\bar{c}\bar{s}]$	2.4852	3.0883	3.0883	3.6917	4.143 [3]
$X_c[cc][\bar{c}\bar{c}]$	6.4680	6.4914	6.4914	6.5204	6.905 [7]

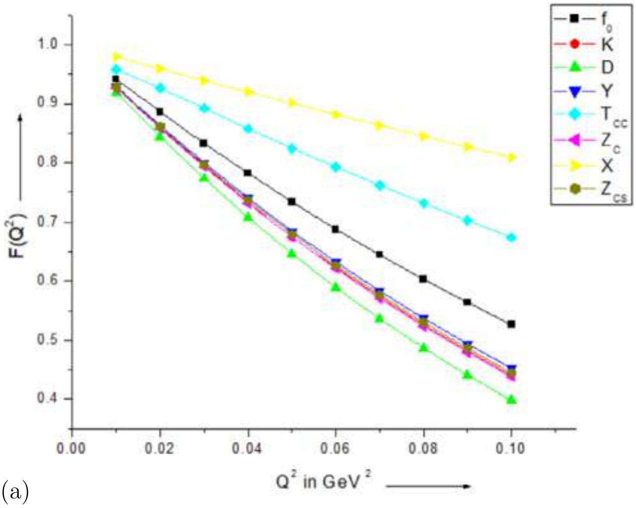
$$|\Psi(r)|^2 = (\frac{8}{\pi^2 r_{12}^6})(r_{12}^2 - r^2)^{3/2}\theta(r_{12} - r) \tag{9}$$

corresponding to the harmonic type of background potential acting between the diquark and the antidiquark. $\theta(r_{12} - r)$ is the usual step function. With the input of the wavefunction in (8), the expression for the form factor is obtained as:

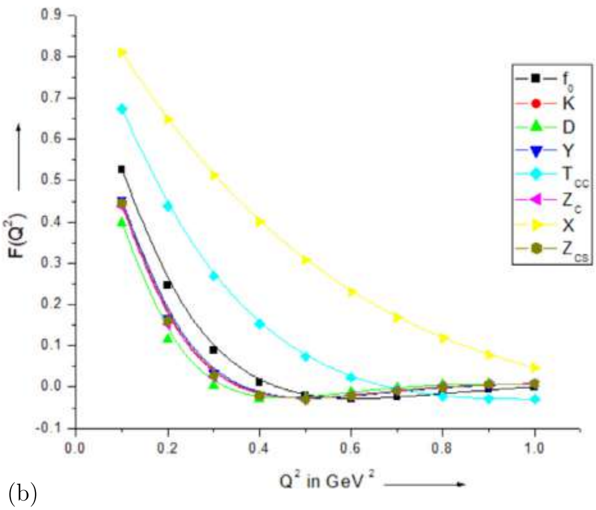
$$F(Q^2) = 48(Qr_{12})^{-3} J_3(Qr_{12}) \tag{10}$$

where $J_3(Qr_{12})$ is the Bessel function of the first kind. The radii of the scalar and vector diquarks depending on their spins have been given input from the existing literature [40–44]. Scalar $[qq]_0$ and vector $[qq]_1$ diquarks are designated as spin 0 and spin 1 constitutions respectively.

We have estimated the form factors $F(Q^2)$ of different tetraquark states comprising scalar and vector diquark-antidiquark for different values of Q^2 in the range of $0 < Q^2 < 0.1$ GeV^2 and $0.1 < Q^2 < 1.0$ GeV^2 separately. The variations of $F(Q^2)$ with Q^2 for different tetraquarks such as $T_{cc}^+[cc\bar{u}\bar{d}]$, $Z_c^-[cd\bar{c}\bar{u}]$, $Z_{cs}^+[cu\bar{c}\bar{s}]$, $K[ud\bar{u}\bar{s}]$, $f_0[ud\bar{u}\bar{d}]$, $D^+[cd\bar{d}\bar{s}]$, $Y[cs\bar{c}\bar{s}]$, $X[cc\bar{c}\bar{c}]$ have been studied in detail and have been displayed in Fig. 1(a), 1(b), 2(a) and 2(b). Fig. 1(a) and 1(b) display the variations of the tetraquarks constituting of scalar diquarks in the two ranges of Q^2 respectively and Fig. 2(a) and 2(b) display the variations of the tetraquarks constituting of vector diquarks in the two ranges of Q^2 respectively.



(a)

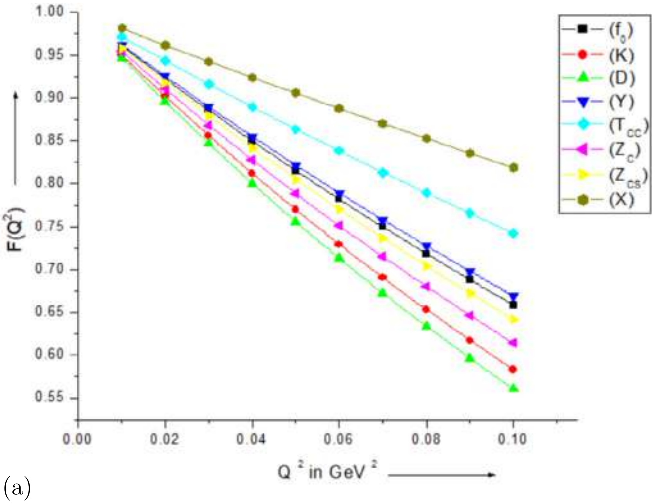


(b)

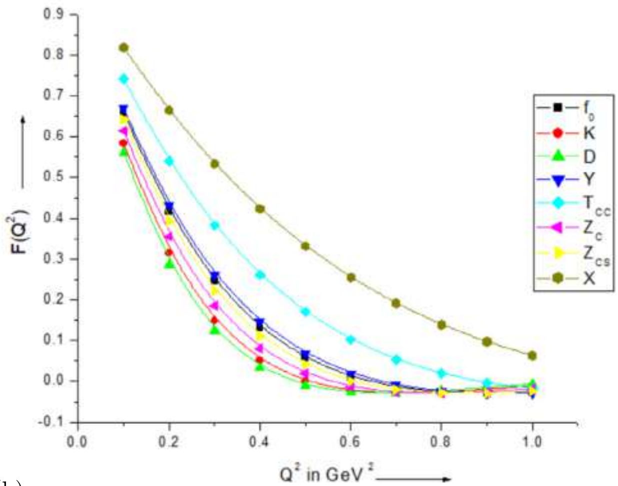
Fig. 1. (a) Variation of the Form factor with Q^2 in the range $0.01 < Q^2 < 0.1$ GeV^2 for the tetraquarks with scalar diquarks. (b) Variation of the Form factor with Q^2 in the range $0.1 < Q^2 < 1.0$ GeV^2 for the tetraquarks with scalar diquarks.

3. Conclusion

In the present work we have estimated the masses of the exotic tetraquarks such as $T_{cc}^{+}[cc\bar{u}\bar{d}]$, $Z_c^{-}[cd\bar{c}\bar{u}]$, $Z_s^{+}[cu\bar{c}\bar{s}]$, $K[ud\bar{u}\bar{s}]$, $f_0[ud\bar{u}\bar{d}]$, $D^{+}[cd\bar{d}\bar{s}]$, $Y[cs\bar{c}\bar{s}]$, $X[cc\bar{c}\bar{c}]$ in the concept of Composite Fermion quasiparticle approach and also studied their form factors in detail. Comparing our computed masses with the experimental data in the Table 2, we have found that the masses of spin 0 configuration of $f_0[ud][\bar{u}\bar{d}]$, $T_{cc}^{+}[cc\bar{u}\bar{d}]$ are close to the experimental values whereas masses of spin 2 configuration of $D^{+}[cd\bar{d}\bar{s}]$, $Z_c^{-}[cd\bar{c}\bar{u}]$, $Z_s^{+}[cu\bar{c}\bar{s}]$, $Y[cs\bar{c}\bar{s}]$, $X[cc\bar{c}\bar{c}]$ are in favourable agreement with the experimental values. It can be stated that the vector-vector



(a)



(b)

Fig. 2. (a) Variation of the Form factor with Q^2 in the range $0.01 < Q^2 < 0.1$ GeV^2 for the tetraquarks with vector diquarks. (b) Variation of the Form factor with Q^2 in the range $0.1 < Q^2 < 1.0$ GeV^2 for the tetraquarks with vector diquarks.

diquark-antidiquark combinations are most preferable. Other tetraquark masses are found to be within the limits of the experimental data. However, it may be stated that we have found the mass splittings of scalar and vector combinations of Z_s, Z_c and Y states quite large and have estimated these deviations as 26% 22% 29% for Z_s, Z_c and Y particles respectively. It can be explained that the exotic tetraquark systems have complicated structure where the diquarks have been described as composite fermions. The formulation depends on the spatial extension which largely manifests the interaction of the system.

The current investigation shows that the quasiparticle CF picture of diquarks reproduces the tetraquark masses quite well. However, we have obtained the mass of [us] diquark smaller than the [ud] diquark mass. It may be mentioned that the uncertain parameters used in our model are the interaction parameter and the radii of different diquarks taken from current literature. The exact value of radius parameter is not known. This may insert some uncertainty in the diquark mass and attributed as the size effect or spatial extension of diquark. Size of the particles has important effect on dynamics. However, it may be noted that the computed masses of the corresponding tetraquarks containing [us] diquarks agree more or less well with the experimental values. We have estimated the percentage variation of tetraquark masses due to small variation of the radius parameter. It has been found that for a variation of ± 0.2 GeV^{-1} in radius, the percentage error in the tetraquark masses varies from 2.17% to 2.36%. Moreover, the mass gaps between the vector and the scalar diquarks for the light-light and heavy-heavy systems are found relatively small, while for the heavy-light systems are large. This can be explained as the input of $\Lambda = 0.573$ GeV [46] for light systems and $\Lambda = 0.6533$ GeV [47] for heavy systems describe the diquark systems well, whereas the heavy-light combined systems are not so well described.

The form factor variations have been displayed in the Figs. 1(a), 1(b) and 2(a) and 2(b). Fig. 1(a) and Fig. 2(a) display the variations of $F(Q^2)$ with Q^2 in the range of $0.01 < Q^2 < 0.1$ GeV^2 for the scalar ($[qq]_0[\bar{q}\bar{q}]_0$) and vector ($[qq]_1[\bar{q}\bar{q}]_0$)/($[qq]_0[\bar{q}\bar{q}]_1$)/ ($[qq]_1[\bar{q}\bar{q}]_1$) tetraquark states respectively. The form factors are found to be falling sharply in the range indicating a clear violation of scaling at lower values of momentum transfers. The effect may be attributed to the interaction of quarks by the exchange of gluons. Fig. 1(b) and Fig. 2(b) represent the variations of the form factors for the momentum transfers in the range of $0.1 < Q^2 < 1.0$ GeV^2 for the scalar and vector tetraquark states respectively. The form factors show a fast decrease in the range of $0.1 < Q^2 < 1.0$ GeV^2. It is interesting to note that the Fig. 1(b) and Fig. 2(b) in the range $0.1 < Q^2 < 1.0$ GeV^2 show $F(x) \sim 1/x^2$ behaviour. According to the constituent counting rule, electromagnetic form factor $\sim s^{2-N}$ where s is Manderstam variable and N is the number of constituent quarks. For the tetraquark state with $N = 4$, our results seem to obey the counting rule in the range $0.1 < Q^2 < 0.1$ GeV^2. It is suggested that the larger the number of constituents, smaller is the chance that the whole system recoils along with the struck constituent. Hence the elastic form factor dies faster with Q^2 as N becomes larger and larger [54,55]. At high energy, the cross section at large scattering angle of a hard exclusive process falls off as a power of the Manderstam variable s. Recently Guo et al. [56] have studied the constituent counting rule in exotic hadrons and observed that for exotic hadrons, in the productions of the X(3872) and others in hard exclusive processes, the results may not be obtained from the naive constituent counting rule. Form factor behaviour in the Fig. 1(b) and Fig. 2(b) show an indication of independence of the momentum transfers in the range beyond $Q^2 > 1$ GeV^2 which may be related to the scaling behaviour. Scaling strongly suggests that experimentally observed strongly interacting particles (hadrons) behave as collections of point-like constituents when probed at high energies. At high energy with improved resolution scale, scaling implies independence of the absolute resolution of the scale and effectively a point like substructure. At large energies and momentum transfers, the cross section depends on one variable only as the photon ceases to scatter coherently off the hadron but solely sees the individual as point-like partons.

The Composite Fermion model describing the exotic tetraquark states is found to be not far from reality. We have used the linear type of confinement potential acting between the diquark and the antidiquark. The kinetic energy or total energy related to the angular momentum or orbital motion of them is not considered in the current investigation. Coulomb potential between diquark and anti-diquark expresses the asymptotic nature of the strong interactions at short distances. It plays a feeble role at this scale (fm). However, we will investigate the diquark-antidiquark system with Coulomb potential and other kind of potentials in our future works to take into account the relevant contributions. The current investigation specially yields good results for the heavy tetraquark systems. But the mass splittings of Z_s, Z_c and Y states are found to be comparatively large. The study of the properties like form factors is very important at this conjecture to reveal the internal structure and dynamics of the systems. In the current investigation it is observed that the newly discovered exotic hadrons show a scaling violation in the low momentum transfers. Only future experiments on these aspects would enlighten us to realize the structure and dynamics of these exotic states.

CRediT authorship contribution statement

The formulation idea has been proposed by B. Chakrabarti and A. Bhattacharya. Execution of the formulation has been done by S. Pal. Overall the manuscript has been compiled by B. Chakrabarti.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

References

[1] Belle Collaboration, S.K. Choi, S.L. Olsen, K. Abe, T. Abe, et al., Observation of a narrow charmoniumlike state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays, Phys. Rev. Lett. 91 (2003) 262001, https://doi.org/10.1103/PhysRevLett.91.262001.

[2] Belle Collaboration, K. Abe, T. Abe, I. Adachi, et al., Observation of a near-threshold $\omega J/\psi$ mass enhancement in exclusive B → K$\omega J/\psi$ decays, Phys. Rev. Lett. 94 (2005) 182002, https://doi.org/10.1103/PhysRevLett.94.182002.

[3] CDF Collaboration, T. Aaltoneon, et al., Evidence for a narrow near-threshold structure in the $J/\psi\phi$ mass spectrum in $B^{+} \to J/\psi\phi K^{+}$ decays, Phys. Rev. Lett. 102 (2009) 242002, https://doi.org/10.1103/PhysRevLett.102.242002;
T. Aaltoneon, et al., Observation of the Y(4140) structure in the $J/\psi\phi$ mass spectrum in $B^{\pm} \to J/\psi\phi K^{\pm}$ decays, Mod. Phys. Lett. A 32 (2017) 1750139, https://doi.org/10.1142/S0217732317501395.

[4] Belle Collaboration, S.K. Choi, S.L. Olsen, K. Abe, T. Abe, et al., Observation of a resonance like structure in the $\pi^{\pm}\psi'$ mass distribution in exclusive $B \to K\pi^{\pm}\psi'$ decays, Phys. Rev. Lett. 100 (2008) 142001, https://doi.org/10.1103/PhysRevLett.100.142001.

[5] Belle Collaboration, K. Chilikin, et al., Observation of a new charged charmoniumlike state in $\bar{B}^{0} \to J/\psi K^{-}\pi^{+}$ decays, Phys. Rev. D 90 (2014) 112009, https://doi.org/10.1103/PhysRevD.90.112009.

[6] Belle Collaboration, A. Bonder, et al., Observation of two charged bottomonium like resonances in $\Upsilon(5S)$ decays, Phys. Rev. Lett. 108 (2012) 122001, https://doi.org/10.1103/PhysRevLett.108.122001.

[7] LHCb Collaboration, R. Aaij, et al., Observation of structure in the J/ψ-pair mass spectrum, Sci. Bull. 65 (2020) 1983, https://doi.org/10.1016/j.scib.2020.08.032.

[8] LHCb collaboration, R. Aaij, et al., Observation of $J/\psi\phi$ structures consistent with exotic states from amplitude analysis of $B^{+} \to J/\psi\phi K^{+}$ decays, Phys. Rev. Lett. 118 (2017) 022003, https://doi.org/10.1103/PhysRevLett.118.022003;
R. Aaij, et al., Amplitude analysis of $B^{+} \to J/\psi\phi K^{+}$ decays, Phys. Rev. D 95 (2017) 012002, https://doi.org/10.1103/PhysRevD.95.012002.

[9] R.L. Jaffe, Multiquark hadrons. I. Phenomenology of $Q^{2}Q^{-2}$ meson, Phys. Rev. D 15 (1977) 267, https://doi.org/10.1103/PhysRevD.15.267.

[10] Y. Iwasaki, A possible model for new resonances: exotics and hidden charm, Prog. Theor. Phys. 54 (1975) 492, https://doi.org/10.1143/PTP.54.492;
Y. Iwasaki, Is a state $c\bar{c}c\bar{c}$ found at 6.0 GeV?, Phys. Rev. Lett. 36 (1976) 1266, https://doi.org/10.1103/PhysRevLett.36.1266.

[11] V.R. Debastiani, F.S. Navarra, A non-relativistic model for the $[cc][\bar{c}\bar{c}]$ tetraquark, Chin. Phys. C 43 (2019) 013105, https://doi.org/10.1088/1674-1137/43/1/013105.

[12] W. Chen, H.X. Chen, X. Liu, T.G. Steele, S.L. Zhu, Hunting for exotic doubly hidden-charm/bottom tetraquark states, Phys. Lett. B 773 (2017) 247–251, https://doi.org/10.1016/j.physletb.2017.08.034.

[13] G.J. Wang, L. Meng, S.L. Zhu, et al., Spectrum of the fully-heavy tetraquark state $QQ\bar{Q}'\bar{Q}'$, Phys. Rev. D 100 (2019) 096013, https://doi.org/10.1103/PhysRevD.100.096013.

[14] W. Park, S. Noh, S.H. Lee, Masses of the doubly heavy tetraquarks in a constituent quark model, Nucl. Phys. A 983 (2019) 1–19, https://doi.org/10.1016/j.nuclphysa.2018.12.019.

[15] S.S. Aagev, K. Azizi, B. Barsbay, H. Sundu, The doubly charmed pseudoscalar tetraquarks $T^{++}_{cc;\bar{s}\bar{s}}$ and $T^{++}_{cc;\bar{d}\bar{s}}$, Nucl. Phys. B 939 (2019) 130–144, https://doi.org/10.1016/j.nuclphysb.2018.12.021.

[16] Z-Y. Di, Z-G. Wang, J-X. Jhang, G-L. Yu, Scalar hidden-charm tetraquark states with QCD sum rules, Commun. Theor. Phys. 69 (2018) 191–202, https://doi.org/10.1088/0253-6102/69/2/191.

[17] J. Vijande, A. Valcarce, N. Barnea, Hyperspherical harmonic formalism for tetraquarks, Int. J. Mod. Phys. A 22 (2007) 561–565, https://doi.org/10.48550/arxiv-hep-ph/0610124.

[18] N. Barnea, J. Vijande, A. Valcarce, Four-quark spectroscopy within the hyperspherical formalism, Phys. Rev. D 73 (2006) 054004, https://doi.org/10.1103/PhysRevD.73.054004.

[19] B. Chakrabarti, A. Bhattacharya, S. Mani, On some properties of di-hadronic states, Phys. Scr. 79 (2009) 025103, https://doi.org/10.1088/0031-8949/79/02/025103.

[20] BaBar Collaboration, A. Sanchez, et al., Evidence for the decay $X(3872)\rightarrow J/\psi\omega$, Phys. Rev. D 82 (2010) 011101, https://doi.org/10.1103/PhysRevD.82.011101.

[21] BaBar Collaboration, B. Aubert, et al., Measurement of the $e^+e^-\rightarrow b\bar{b}$ cross section between $\sqrt{s}=10.54$ and 11.20 GeV, Phys. Rev. Lett. 102 (2009) 012001, https://doi.org/10.1103/PhysRevLett.102.012001.

[22] LHCb Collaboration, R. Aaij, et al., Evidence for the decay $X(3872)\rightarrow\psi(2S)\Upsilon$, Nucl. Phys. B 886 (2014) 665, https://doi.org/10.1016/j.nuclphysb.2014.06.011.

[23] G. Sterman, P. Stoler, Hadronic form factors and perturbative QCD, Annu. Rev. Nucl. Part. Sci. 47 (1997) 193–233, https://doi.org/10.1146/annurev.nucl.47.1.193.

[24] C. Alexandrou, Hadron structure and form factors, Proc. Sci. Lattice 2010 (2010) 001, https://inspirehep.net/files/5b5f1bf7237e925af16d2fcfa7bdf956.

[25] A. Khodjamirian, Hadron Form Factors: From Basic Phenomenology to QCD Sum Rules, CRC Press, New York, ISBN 9781138306752, 2020.

[26] R.N. Faustov, V.O. Galkin, E.M. Savchenko, Heavy tetraquarks in the relativistic quark model, Universe 7 (2021) 94, https://doi.org/10.3390/universe7040094.

[27] S.S. Agaev, K. Azizi, B. Barsday, H. Sundu, A family of double-beauty tetraquarks: axial-vector state $T^{-}_{bb;\bar{u}\bar{s}}$, Chin. Phys. C 45 (2021) 013105, https://doi.org/10.1088/1674-1137/1bc16d.

[28] M.A. Durnev, Form factors of tetraquarks, Russ. Phys. J. 53 (2010) 311–315, https://doi.org/10.1007/s11182-010-9417-y.

[29] Q.F. Lü, D.Y. Chen, Y.B. Dong, Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model, Phys. Rev. D 102 (2020) 034012, https://doi.org/10.1103/PhysRevD.102.034012.

[30] H. Mutuk, Nonrelativistic treatment of fully-heavy tetraquarks as diquark-antidiquark states, Eur. Phys. J. C 81 (2021) 367, https://doi.org/10.1140/epjc/s10052-021-09176-8.

[31] Q. Qin, Y-F. Shen, F-S. Yu, Discovery potentials of double-charm tetraquarks, Chin. Phys. C 45 (2021) 103106, https://doi.org/10.1088/1674-1137/acIb97.

[32] A. Francis, R.J. Hudspith, R. Lewis, K. Maltman, Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD, Phys. Rev. D 99 (2019) 054505, https://doi.org/10.1103/PhysRevD.99.054505.

[33] W. Lucha, D. Melikhov, H. Sazdjian, Cluster reducibility of multiquark operators and tetraquark-adequate QCD sum rules, Rev. Mex. Fis. Suppl. 3 (2022) 0308035, https://doi.org/10.488550/j.arXiv.2122.10239.

[34] S.L. Olsen, T. Skwarnicki, D. Zieminska, Nonstandard heavy mesons and baryons: experimental evidence, Rev. Mod. Phys. 90 (2018) 015003, https://doi.org/10.1103/RevModPhys.90.015003.

[35] F.K. Guo, et al., Hadronic molecules, Rev. Mod. Phys. 90 (2018) 015004, https://doi.org/10.1103/RevModPhys.90.015004, Erratum: Rev. Mod. Phys. 94 (2022) 029901.

[36] H.X. Chen, W. Chen, X. Liu, S.L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rep. 639 (2016) 1–121, https://doi.org/10.1016/j.physrep.2016.05.004.

[37] A.R. Chari, F.W.M. Haldane, K. Yang, A gauge invariant way to evaluate quasiparticle effective mass in fermionic systems with gauge interactions, arXiv:cond-mat/9707055v1, 1997.

[38] A. Bhattacharya, B. Chakrabarti, S.N. Banerjee, On some properties of the ratio of matrix elements $|\frac{V_{ub}}{V_{cb}}|$ related to the Fermi momentum (p_f), Eur. Phys. J. C 2 (1998) 671–673, https://doi.org/10.1007/s100529800741.

[39] A. Bhattacharya, B. Chakrabarti, T. Sarkhel, S.N. Banerjee, Dependence of $|V_{cb}|$ on Fermi momentum, Int. J. Mod. Phys. A 15 (2000) 2053–2058, https://doi.org/10.1142/S0217751X00000835.

[40] A.S. de Castro, H.F. de Carvalho, A.C.B. Antunes, A diquark model for baryon spectroscopy, Z. Phys. C 57 (1993) 315–317, https://doi.org/10.1007/BF01565063.

[41] B. Chakrabarti, B and D meson decay constants revisited, Mod. Phys. Lett. A 12 (28) (1997) 2133–2140, https://doi.org/10.1142/S0217732397002181.

[42] K. Nagata, A. Hosaka, Structure of the nucleon and Roper resonance with diquark correlations, Mod. Phys. Lett. A 23 (2008) 2397–2400, https://doi.org/10.1142/S0217732308029460.

[43] A. Bhattacharya, A. Sagari, B. Chakrabarti, S. Mani, Magnetic moments of baryons and exotics in quasi particle diquark model, in: 18th International Spin Physics Symposium, Virginia, in: AIP Conference Proceedings, vol. 1149, 2008, p. 593.

[44] D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Mass spectra of doubly heavy baryons in the relativistic quark model, Phys. Rev. D 66 (2022) 014008, https://doi.org/10.1103/PhysRevD.66.014008.

[45] M. Karliner, H.J. Lipkin, A diquark–triquark model for the KN pentaquark, Phys. Lett. B 575 (2003) 249–255, https://doi.org/10.1016/j.physletb.2003.09.062.

[46] S. Pepin, M.C. Birse, J.A. McGovern, N.R. Walet, Nucleons or diquarks: competition between clustering and color superconductivity in quark matter, Phys. Rev. C 61 (2000) 055209, https://doi.org/10.1103/PhysRevC.61.055209.

[47] M. Huang, P. Zhuang, W. Chao, Charge neutrality effects on two-flavor color superconductivity, Phys. Rev. D 67 (2003) 065015, https://doi.org/10.1103/PhysRevD.67.065015.

[48] W. Lucha, F.F. Schoberl, D. Gromes, Bound states of quarks, Phys. Rep. 200 (1991) 127–240, https://doi.org/10.1016/0370-1573(91)90001-3.

[49] E. Swanson, New particle hints at four-quark matter, Physics 6 (2013) 69, https://physics.aps.org/articles/v6/69.

[50] LHCb Collaboration, R. Aaij, et al., Observation of new resonances decaying to $J/\psi K^+$ and $J/\psi\phi$, Phys. Rev. Lett. 127 (2021) 082001, https://doi.org/10.1103/PhysRevLett.127.082001.

[51] A. Zhang, Regge-trajectory analysis of $D^*_{SJ}(2317)^{+-}$,$D_{SJ}(2460)^{+-}$ and $D_{SJ}(2632)^+$ mesons, Phys. Rev. D 72 (2005) 017902, https://doi.org/10.1103/PhysRevD.72.017902.

[52] LHCb collaboration, R. Aaij, et al., Observation of an exotic narrow doubly charmed tetraquark, Nat. Phys. (2022), arXiv:2109.01038 [hep-ex].

[53] Y.R. Liu, H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Pentaquark and tetraquark states, Prog. Part. Nucl. Phys. 107 (2019) 237–320, https://doi.org/10.1016/j.ppnp.2019.04.003.

[54] S.J. Brodsky, B.T. Chertok, Asymptotic form factors of hadrons and nuclei and the continuity of particle and nuclear dynamics, Phys. Rev. D 14 (1976) 3003, https://doi.org/10.1103/PhysRevD.14.3003.

[55] F.E. Close, Quarks and Partons, Academic Press, Cambridge, 1981, p. 315.

[56] F.K. Guo, U.G. Meißner, W. Wang, On the constituent counting rule for hard exclusive processes involving multi-quark states, Chin. Phys. C 41 (2017) 053108, https://doi.org/10.1088/1674-1137/41/5/053108.