Journal of Fractional Calculus and Applications
Vol. 14(1) Jan. 2023, pp. 21-25.
ISSN: 2090-5858.
http://math-frac.org/Journals/JFCA/

CHARGE OF A QUANTUM PARTICLE AS IMAGINARY ENERGY AND FRACTAL POTENTIAL ENERGY

A.BHATTACHARYA, R. GHOSH, S.PAL, D.S. BHATTACHARYA

Abstract. The charge of a quantum particle is suggested to have a contribution from fractal potential energy which arises from the non differentiability of space-time along with imaginary mass. It has been suggested from scale relativity that quantum particle are subjected to a fractal space-time which predicts two valuedness in the velocity field. The velocity contains a divergent term which in turn depends on the fractal dimension of the particle. The divergent part has been extracted for hadron which are described as fractal objects with fractal dimension of 9/2 in the context of statistical model. A contribution from fractal potential has been suggested and an expression for the charge is extracted with imaginary fractal potential energy which is found to depend on the covariant of velocity field. It is suggested that the fractal potential manifests the charge of the quantum particles and closely related to the space-time of the quantum particle which is fractal.

1. Introduction

The existence of imaginary mass and energy are the one of the area of interest for a long time. The particles with imaginary masses but with real energy and momentum is suggested to exist [1]. The imaginary energy usually describes the time dependent part of the wave function and the life time of states. It is wellknown that the fundamental properties of a particle are electric charge and mass. The gravitational and electromagnetic interaction can be visualized as the interactions of their complex energies. The electric charges usually understood as the interaction among the charges in the framework of coulomb interaction whereas the mass as the gravitational interaction [2-4]. It has been suggested that the electric charge is a pack of certain amount of imaginary energy and quantization of the imaginary energy gives electric charge quanta. Zhang [5] has investigated electric charge as a form of imaginary energy and suggested that classically it unifies gravitational and electric forces.

The dimensionality of a quantum system is a very important property which governs the dynamics of the system itself. Non differentiability of space time of quantum particles and the dimensionality relating the fractional charges of particles

2010 *Mathematics Subject Classification.* 34A12, 34A30, 34D20.
Key words and phrases. Fractal space, Fractional Charge, Imaginary mass, Hadrons.
Submitted Dec. 12 2021. Revised April 8, 2022.

are investigated by a number of authors [6-13]. It has been suggested that the scale relativity is much more fundamental and considered to manifest the fact that set has has structure at every minute level. Nottale [7] has pointed out that the resolution at which a system is observed should be considered with motion at the same footing. The path of the quantum particle is fractal and uncertainty is basically associated with the space-time structure of the quantum particles [10]. The scale relativity deals with the passage from differential to non-differential geometry where time derivative of displacement is replaced by covariant or making the system covariant. The non differentiability includes two valuedness in the velocity and a complex fractal force is identified due to fractal fluctuation [7].

In the current work we have investigated the charges of a particle as imaginary energy. The contribution of fractal potential energy which originates from the fractal space-time of the quantum particle has been included in the energy expression of a particle's total energy. Divergent part of the velocity field is extracted for hadrons with fractal dimension D_F =9/2 which is a prediction of statistical model [14]. An expression for the electric charge has been extracted with contribution from fractal potential.

2. Formulation

To describe the motion of a quantum particle in fractal space, the complete geodesic equation includes average distance and stochastic fluctuation which is closely related to the fractal dimension D_F as: [7].

$$d\xi = a\sqrt{2D}(dt^2)^{\frac{1}{2D_F}} \tag{1}$$

where $< a^2 >$ =1, D = $\frac{\hbar}{2m}$ or 2mD = $\hbar$, a fundamental constant. The corresponding velocity possesses two valuedness which can be expressed as:[7],

$$dX/dt = v + a\sqrt{2D}(dt^2)^{\frac{1}{2D_F}}(dt^2)^{\frac{-1}{2}} \tag{2}$$

$$dX/dt = v + w \tag{3}$$

which indicates that the velocity contains a divergent term (w). For last few decades we have suggested a statistical model for hadron which is widely used to study the properties of hadrons and exotics [14-17].The most important prediction of the statistical model is describing a hadron as a fractal object and prediction of fractal dimension of a hadron. The wave function for hadron is obtained as:[14]

$$\psi(r) = A(r_0 - r)^{3/4}\theta(r_0 - r) \tag{4}$$

where r_0 is the typical radius parameter of a hadron and A is a constant. The wave function has a remarkable feature that it is non analytic in nature and shows a scaling property. The fractal dimension for hadrons has been found to be D = 9/2 [13-15] when the topological dimension D_T = 3. From (2) with D_F =9/2, a=1, we arrive at :

$$w = \frac{\hbar}{\sqrt{m}}(dt^2)^{\frac{-7}{18}} \tag{5}$$

which diverges as dt→0 and exhibits its non differentiability as a consequence of the fractal nature of the hadron. The fractal dimension indicates the measure of complexity of a system which can not be shown by the integer dimensions. It has been pointed out by Nottale [7] that each of v and w is of two values due to

infinitesimal time symmetry (dt→-dt)and suggested complete velocity field which includes differentiable and fractal part as:

$$\tilde{\nu} = (\frac{v_+ + v_-}{2} - i\frac{v_+ + v_-}{2}) + (\frac{w_+ + w_-}{2} - i\frac{w_+ + w_-}{2}) \tag{6}$$

and the complex Lagrangian runs as:

$$L = \frac{1}{2}m\tilde{\nu}^2 \tag{7}$$

$$L = \frac{1}{2}m(v + w)^2 \tag{8}$$

The effect of non differentiable fractal fluctuation can be described by an effective force and a fractal potential can be derived [7]. It has been stated that in the scale relativistic approach force is nothing but the manifestation of every structure of the space. An expression for fractal potential has been derived as: [7]

$$\phi_F = -imD\nabla\nu \tag{9}$$

The above equation shows that the non differentiable space-time and fractal nature of the space, induce a fractal part of the potential which originates from two valuedness of the velocity field of the particle.

Electric charge is basic property of matter carried by elementary particles. Understanding of electric charge as a form of imaginary energy is area of interest for long. The energy of a particle is suggested to be a complex energy where the real part represents mass and the imaginary part represents the electric charge [5]. The total energy can be expressed as:

$$E = E^M + iE^Q \tag{10}$$

where E^M is gravitational mass $E^M = \mathrm{M}c^2$ and $E^Q = \frac{Q}{\sqrt{G}}c^2$, Q is the electric charge of the particle, G is the gravitational constant. We suggest that the expression of energy of a quantum particle in (10) will be modified by the contribution from the fractal potential as obtained in (10) and can be recast as:

$$E = E^M + iE^Q - imD\nabla\nu \tag{11}$$

with $2\mathrm{mD} = \hbar$, we get;

$$E = E^M + i[E^Q - (\frac{\hbar}{2})\nabla\nu] \tag{12}$$

$$E^{Q'} = i(E^Q - mD\nabla\nu) \tag{13}$$

For antiparticle with imaginary energy negative sign the energy expression will be:

$$E = E^M + i(mD\nabla\nu - E^Q) \tag{14}$$

From (14) we found that the imaginary part of the total energy is modified from the contribution of fractal potential and found to depend on the covariant of complex velocity. The charge of a particle which is described as the imaginary part of the energy suggested to have a contribution from space-time structure of the particle itself.

3. RESULTS AND CONCLUSIONS

In the current work we have investigated the charge of a quantum particle as imaginary energy considering the contribution due to fractal space-time of the quantum particles. Divergent part of the velocity field is related to the fractal dimension and we have derived an expression for divergent part of the velocity field for hadron with the input of fractal dimension of hadron 9/2, predicted from statistical model. The origin of charge is one of the fundamental question which is understood as the imaginary energy of a particle. Ahanger [18] has investigated complex matter space and special theory of relativity in a unifying approach and discussed the relativistic approach of energy and momentum in complex matter space (CMS). They have made a detail study of CMS with the postulates that mass and charge is two intrinsic component of matter where real parts equals to the mass and imaginary part as the charge and in one dimension space, position and velocity are real numbers. They have pointed out that CMS will give better understanding of energy momentum conservation, antiparticle, tachyon, dark energy, dark matter and also unification of forces. Theory of relativity admits particle with imaginary mass and particle moving with velocity greater than light but only prohibited by the law of causality. Current work focusses on the consequences of fractal space-time to the charge of a quantum particle. The two valuedness of the velocity which are fractal function of resolution has been discussed. The charge of the particle is modified with contribution from fractal potential. It is interesting to observe in the current investigation that the complex velocity field is related to the space-time geometry and fundamental property, the charge of a particle. The charge is suggested to be manifestation of fractal space-time in addition to the imaginary mass E^Q. It is suggested that the detailed study of the space time structure is unavoidable for our understanding of fundamental laws of nature and the properties of the particles. Recently we have investigated the fractional charge of quarks relating fractal dimension describing quarks as quasi particle [19]. In our future work we focus on the fractional charges of quarks related to imaginary mass.

REFERENCES

[1] R. Fox, "Possible Existence of Particles of Imaginary Mass, Energy and Momentum", Nature Physical Science,Vol. 232 129-130, 1971.
[2] I. Newton, "Mathematical Principle of Natural Philosophy",(1687), Third Edition.
[3] P.M.A.Dirac, "The Quantum Theory of Electrons", Proceedings of Royal Society A, Vol. 117, 610-624, 1928.
[4] A. Einstein, "Zur Elektrodynamik bewegter Krper", Annals of Physics, Vol. 322,891-921, 1905.
[5] T. Zhang, "Electric Charge as Form of Imaginary Energy", Progress in Physics,Vol. 2, 79-83, 2008.
[6] B.B. Mandelbrot, "Les Objets Fractals : forme, hasard et dimension", Paris, Flammarion 1975.
[7] L. Nottale, "Scale Relativity and Fractal Space-Time", World Scientific,197, 2011.
[8] L.Nottale, "On the Transition from the Classical to Quantum Regime in Fractal Space-Time Theory", Chaos, Solitons and Fractals,Vol. 25,797-803 2005.
[9] R. P.Feynmam, "Quantum Mechanics and Path Integrals",MacGrow-Hill, New York.1965.
[10] A. Bhattacharya, R. Saha and B. Chakrabarty, "Fractal Space Time and Variation of the Fine Structure Constant", European Physical Journal Plus, Vol.127,1-6, 2012.
[11] H. Kroger,"Fractal Geometry in Quantum Mechanics, Field Theory and Spin Systems", Physics Reports,Vol.323, 81-181 2000.

[12] A. Deppman and E. Magias, "Fractal Structure in Gauge Field", Physics,Vol.1, 103-110, 2019.
[13] R.Saha, R. Ghosh, D.S. Bhattacharya, A. Bhattacharya, "Fractal Properties and Fractional Multipole Moments of Hadron", AIP Conference Proceedings, Vol. 2027,020013, 2019.
[14] S. N.Banerjee, A. Bhattacharya, B. Chakrabarti, S. Banerjee, "The Properties of Hadron", International Journal of Modern Physics A, Vol.16(2), 201-207, 2001.
[15] S. N. Banerjee, A. Bhattacharya, B. Chakrabarti, S. Banerjee, "On Some Properties of Meson", International Journal of Modern Physics A,Vol. 17(32), 4939-4945, 2002.
[16] A. Bhattacharya, S.N. Banerjee, "Binding Energy of Proton as a Quarkonium System in the Statistical Model", Progress of Theoretical Physics,Vol. 77(1), 16-19, 1987.
[17] A. Bhattacharya, B.Chakrabarti, S.N. Banerjee, "On Some properties of the ratio of the matrix elements $|\frac{V_{ub}}{V_{cb}}|$ related to the Fermi momentum p_f, Euro. Phys.J C. Vol. 2, 671-673, 1998.
[18] R. Ahangar, "Foundation of Complex Matter Space and Special Theory of Relativity, a Unifying Approach", Journal Of Nuclear and Particle Physics, Vol.4,147-153,2014.
[19] A. Bhattacharya, S. Pal, D.S. Bhattacharya, P. Dhara, "Fractional Charges of Quarks and Fractal Properties of Hadrons", Journal of Particle Physics, Vol. 5, 1-4, 2021.

A. Bhattacharya
Department of Physics, Adamas University, Kolkata, India
E-mail address: aparajita.bhattacharya@adamasuniversity.ac.in

R. Ghosh
Jadavpur University University, Kolkata, India
E-mail address: rismita.ghosh@gmail.com

S.Pal
Basanti Devi College, Kolkata , India.
E-mail address: shukalacharya123@gmail.com

D.S. Bhattacharya
INFN, Trieste, Italy.
E-mail address: deb.sankar.bhattacharya@gmail.com